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A POTENTIAL TREATMENT OF HEAT CONDUCTION IN WATER VAPOR WITH TEMPERATURE-DEPENDENT SPECTRUM†

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Abstract—The heat transfer by combined infrared-radiation and conduction in a layer of water vapor is analyzed by using a potential method and measured values of spectral absorption coefficient in the temperature range between 600 and 3000°K. It is found that the problem with the temperature-dependent spectrum can be treated, with good approximation, as one with the spectral absorption coefficient evaluated at a constant reference temperature. Thus, an approximate method of solution for the general problem is established, at least for water vapor. Calculations are also made for a fictitious gas having a very narrow single-band of absorption–emission. Various approximations which have been discussed by previous investigators are re-examined.

NOMENCLATURE

G, g, Green's functions;
h, $3\varepsilon/[2(2-\varepsilon)]$;
I^*_ω, directionally averaged spectral intensity of radiation;
k, k_0, thermal conductivity;
L, physical thickness;
l, $L\varkappa_p(p)$, optical thickness;
l_ω, $\int_0^L \varkappa_\omega(T^*, \omega)\,\mathrm{d}x^*$;
N, $k_0/(4\sigma T_0^{*3}L)$;
N', $k_0\varkappa_p(p)/4\sigma T_0^{*3})$;
n, inward-drawn normal at surface;
p, pressure;
q^*, total heat flux;
q, $q^*/(\sigma T_0^{*4})$;
q_a, defined by (23);
T^*, temperature;
T_0^*, reference temperature;
T, T^*/T_0^*;
x, x^*/L;
x_ω, $\int_0^{x^*} \varkappa_\omega(T^*, \omega)\,\mathrm{d}x^*$;
β_ω, $L\varkappa_\omega(T_0^*, \omega)$;
γ_ω, $\varkappa_\omega(p, \omega)/\varkappa_p(p)$;
ω, wave number;
ε, emissivity of surface;
σ, Stefan–Boltzmann constant;
$\varkappa_p$, Planck mean coefficient defined by (35);
$\varkappa_\omega$, spectral absorption coefficient;
ϕ_ω, spectral radiation potential, $\pi I^*_\omega/\sigma T_0^{*4}$;
Ψ, radiation potential defined by (15) and (28).

Subscripts

b, black body radiation;
1, 2, surfaces at $x^* = 0, L$, except stated otherwise;
0, reference.

INTRODUCTION

THE PROBLEM of heat transfer by combined conduction and radiation in the infrared range has been of great interest in the past few years.

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Gardon [1] studied the transient and steady heat transfer in a plane glass layer for the temperature range between 27 and 707°C. The problem was formulated step-by-step and calculated by a method of finite difference. Howell and Perlmutter [2] investigated the radiative transfer in a hydrogen gas layer by using Monte Carlo Method and line-average absorption coefficients. Gille and Goody [3] studied analytically and experimentally the combined conduction and radiation in an ammonia gas layer through the use of total gas emittance and the method of kernel approximation. Cess *et al.* [4] examined the radiative transfer in a carbon monoxide layer with heat generation. Novotny and Kelleher [5] analyzed the combined radiation and conduction in a carbon dioxide layer. Analyses in [4] and [5] were made by assuming the absorption coefficient as frequency-dependent only and by using the kernel approximation and total band absorptance correlated by Edward and Menard [6] and Tien and Lowder [7].

It was concluded in [4] and [5] that the grey approximation is totally inadequate to predict the heat transfer in gases with spectral-dependent absorption coefficient. This inadequacy was also found in [2] but was not as severe as shown in [4] and [5]. Sparrow and Cess [8] concluded that this discrepancy was due to the extremely wide band of hydrogen gas.

In this paper the heat transfer by conduction and radiation in a water-vapor layer is treated as a potential problem [9]. Measured values of the line-averaged absorption coefficient reported by Ferriso, Ludwig and Thomson [10] are used. The analysis is first made for the general case with spectral- and temperature-dependent absorption coefficient, and then for a simple one with the spectral absorption coefficient evaluated at a given reference temperature. If the temperature fields and heat fluxes obtained for these two cases are in good agreement, then an approximate analysis of the general problem is established, at least for water vapor, and the potential method can be extended for the solution of more complicated problems, such as multi-dimensional heat flow in arbitrarily shaped domains, with temperature-dependent spectrum and variable boundary conditions, as will be discussed later. This is the main purpose of this study.

Calculations are also made for a fictitious gas with a narrow, single absorption–emission band for the purpose of re-examining the inadequacy of the various approximations which were discussed in [4] and [5].

STATEMENT OF PROBLEM

We consider the steady heat transfer in a conducting, absorbing and emitting medium bounded by two infinite parallel plane surfaces. We assume that (i) local thermodynamic equilibrium exists, (ii) scattering of radiation is negligible, (iii) refractive index of the medium is equal to unity, (iv) the bounding surfaces are grey, emit and reflect radiation diffusely, and have the same value of emissivity, and (v) the surfaces at $x^* = 0$ and L are kept at constant temperature T_1^* and T_2^*, respectively, with $T_1^* > T_2^*$.

With the above assumptions, the differential equations and boundary conditions of the spectral intensity and temperature can be obtained in the same manner as those reported in [9] for a grey medium. With notations defined in the Nomenclature, they are

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x^*}\left(\frac{1}{\kappa_\omega}\frac{\mathrm{d}I_\omega^*}{\mathrm{d}x^*}\right) &= 3\kappa_\omega I_\omega^* - 3\kappa_\omega I_{b\omega}^*, \qquad (1)\\ \frac{\mathrm{d}}{\mathrm{d}x^*}\left(k\frac{\mathrm{d}T^*}{\mathrm{d}x^*}\right) &= -4\pi\int_{\omega_1}^{\omega_2}\kappa_\omega(I_\omega^* - I_{b\omega}^*)\,\mathrm{d}\omega \qquad (2)\end{aligned}\right\} \quad 0 < x^* < L$$

$$\frac{dI_\omega^*}{dx^*} = h\varkappa_\omega(I_\omega^* - I_{b\,\omega}^*), \qquad x^* = 0 \tag{3}$$

$$\frac{dI_\omega^*}{dx^*} = -h\varkappa_\omega(I_\omega^* - I_{b\,\omega}^*), \qquad x^* = L \tag{4}$$

$$T^* = T_1^* \qquad x^* = 0 \tag{5}$$

$$T^* = T_2^* \qquad x^* = L \tag{6}$$

where

$$h = \frac{3\varepsilon}{2(2-\varepsilon)} \tag{7}$$

and ω_1 and ω_2 represent the lower and upper wave numbers of the spectrum concerned. An alternative form of (2) may be written as

$$\frac{d}{dx^*}\left(k\frac{dT^*}{dx^*}\right) + \frac{4\pi}{3}\frac{d}{dx^*}\int_{\omega_1}^{\omega_2}\frac{1}{\varkappa_\omega}\frac{dI_\omega^*}{dx^*}d\omega = 0. \tag{8}$$

The thermal conductivity k is, in general, a function of temperature. If we introduce $k_0\theta^* = \int_{T^*} k(T^*)\,dT^*$, the terms on the left-hand side of (2) and (3) become $k_0\nabla^2\theta^*$ where k_0 is a constant having the same dimension of k. Thus, there is no difficulty to take the variation of thermal conductivity with temperature into account. In the following, however, we shall consider k as constant so that results obtained for various cases can be readily compared.

Equation (2) shows that, whenever emission ($\varkappa_\omega I_{b\omega}^*$) is larger than absorption ($\varkappa_\omega I_\omega^*$), radiation produces a distributed heat sink in the medium and the temperature curve will be concave. This will occur in high temperature region. Conversely, wherever emission is smaller than absorption, radiation produces a distributed heat source and the temperature curve will be convex. This will occur in the low temperature region. Therefore, temperature curves will, in general, appear in an S-shape. This holds true both for grey [11] and for nongrey media.

INTEGRAL EQUATIONS FOR THE GENERAL CASE $\varkappa_\omega(T^*, \omega)$

Introducing the new variables x, x_ω, ϕ_ω and T as defined in the Nomenclature, we obtain from (1) through (6)

$$\frac{d^2\phi_\omega}{dx_\omega^2} - 3\phi_\omega = -3\phi_{b\omega} \qquad 0 < x_\omega < l_\omega \tag{9}$$

$$\frac{d\phi_\omega}{dx_\omega} = h[\phi_\omega(0) - \phi_{b\omega 1}] \qquad x_\omega = 0 \tag{10}$$

$$\frac{d\phi_\omega}{dx_\omega} = -h[\phi_\omega(l_\omega) - \phi_{b\omega 2}] \qquad x_\omega = l_\omega \tag{11}$$

$$\frac{d^2T}{dx^2} = -\frac{1}{N}\Psi(x) \tag{12}$$

$$T = T_1 \qquad x = 0 \tag{13}$$

$$T = T_2 \qquad x = 1 \tag{14}$$

where

$$\Psi(x) = \int_{\Delta\omega} L\varkappa_\omega(T^*, \omega)(\phi_\omega - \phi_{b\omega})\,d\omega. \tag{15}$$

Note that the variables x_ω and x as defined in the Nomenclature are related through $T^*(x^*)$.

We wish to solve (9) and (12) by the method of successive approximations. For this purpose, we transform them into integral equations by the use of Green's functions. The Green's function associated with ϕ_ω is known [11],

$$G(x_\omega|x'_\omega, \omega) = \frac{[(\sqrt{3})\cosh(\sqrt{3})x'_\omega + h\sinh(\sqrt{3})x'_\omega][(\sqrt{3})\cosh(\sqrt{3})(l_\omega - x_\omega) + h\sinh(\sqrt{3})(l_\omega - x_\omega)]}{(\sqrt{3})[(3+h^2)\sinh(\sqrt{3})l_\omega + 2(\sqrt{3})h\cosh(\sqrt{3})l_\omega]} \tag{16}$$

for $x_\omega > x'_\omega$. For $x_\omega < x'_\omega$ the results is the same by interchanging x_ω and x'_ω. The Green's function associated with T is also known [11],

$$g(x|x') = x(1 - x') \qquad \text{for } x < x'. \tag{17}$$

The interchange of x and x' gives the result for $x > x'$. The integral equations of ϕ_ω and T are then obtained as follows:

$$\phi_\omega(x_\omega, \omega) = f(x_\omega, \omega) + 3\int_0^{l_\omega} \phi_{b\omega}(x'_\omega, \omega)\, G(x_\omega|x'_\omega, \omega)\, \mathrm{d}x'_\omega \tag{18}$$

where

$$f(x_\omega, \omega) = A(\omega)\sinh(\sqrt{3})x_\omega + B(\omega)\cosh(\sqrt{3})x_\omega \tag{19}$$

$$A(\omega) = \frac{h}{\sqrt{3}}[B(\omega) - \phi_{b\omega 1}(0, \omega)] \tag{20}$$

$$B(\omega) = \frac{h\phi_{b\omega 1}[(\sqrt{3})\cosh(\sqrt{3})l_\omega + h\sinh(\sqrt{3})l_\omega] + (\sqrt{3})h\phi_{b\omega 2}}{(3 + h^2)\sinh(\sqrt{3})l_\omega + 2(\sqrt{3})h\cosh(\sqrt{3})l_\omega}$$

and

$$T = T_1 - (T_1 - T_2)x + \frac{1}{N}\int_0^1 \Psi(x')\, g(x|x')\, \mathrm{d}x' \tag{21}$$

where $\Psi(x)$ is defined by (15) and $\phi_{b\omega}(x_\omega, \omega)$ is related to $\phi_{b\omega}(x, \omega)$ through $x_\omega(T, \omega)$ and $T(x)$.

Once $T(x)$ and $\phi_\omega(x_\omega, \omega)$ are found, the heat flux can be calculated by

$$q = -4N\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right) - \frac{4}{3}\int_{\omega_1}^{\omega_2} \frac{\mathrm{d}\phi_\omega}{\mathrm{d}x_\omega}\mathrm{d}\omega + q_a \tag{22}$$

where q_a is the part of radiant enery transmitted from one surface to the other through the portion or portions of spectra where $\kappa_\omega = 0$ and can be calculated by

$$q_a = \frac{1}{2/\varepsilon - 1}\left[T_1^4 - T_2^4 - \int_{\omega_1}^{\omega_2}(\phi_{b\omega 1} - \phi_{b\omega 2})\,\mathrm{d}\omega\right] \tag{23}$$

INTEGRAL EQUATIONS FOR THE CASE $\kappa_\omega(T_0^*, \omega)$

If the absorption coefficient is assumed as dependent only on the frequency or wave number, (1), (8), (3) and (4) become

$$\left.\begin{aligned} &\frac{\mathrm{d}^2\phi_\omega}{\mathrm{d}x^2} - 3\beta_\omega^2\phi_\omega = -3\beta_\omega^2\phi_{b\omega} && (24)\\ &\frac{\mathrm{d}^2T}{\mathrm{d}x^2} + \frac{1}{3N}\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} = 0 && (25)\end{aligned}\right\} \quad 0 < x < 1$$

$$\frac{\mathrm{d}\phi_\omega}{\mathrm{d}x} = h\beta_\omega(\phi_\omega - \phi_{b\omega 1}) \qquad x = 0 \tag{26}$$

$$\frac{\mathrm{d}\phi_\omega}{\mathrm{d}x} = -h\beta_\omega(\phi_\omega - \phi_{b\omega 2}) \qquad x = 1 \tag{27}$$

where

$$\Psi(x) = \int_{\omega_1}^{\omega_2} \frac{\phi_\omega}{\beta_\omega}\,\mathrm{d}\omega. \tag{28}$$

The boundary conditions on T are the same as those in (13) and (14). The Green's function associated with ϕ_ω for this case is

$$G(x|x', \omega) = \frac{[(\sqrt{3})\cosh(\sqrt{3})\beta_\omega x' + h\sinh(\sqrt{3})\beta_\omega x']\,[(\sqrt{3})\cosh(\sqrt{3})\beta_\omega(1 - x) + h\sinh(\sqrt{3})\beta_\omega(1 - x)]}{(\sqrt{3})\beta_\omega[(3 + h^2)\sinh(\sqrt{3})\beta_\omega + 2(\sqrt{3})h\cosh(\sqrt{3})\beta_\omega]} \tag{29}$$

for $x > x'$. For $x < x'$, we simply interchange x and x' in (29).

The integral equation of ϕ_ω is then

$$\phi_\omega(x, \omega) = f(x, \omega) + 3\beta_\omega^2 \int_0^1 \phi_{b\omega}(x', \omega)\, G(x|x', \omega)\, dx' \quad (30)$$

where

$$f(x, \omega) = A(\omega) \sinh(\sqrt{3})\beta_\omega x + B(\omega) \cosh(\sqrt{3})\beta_\omega x \quad (31)$$

each tip. This arbitrarily assigned value will affect considerably the absolute value of $\Psi(x)$, but it will affect very little the value of $\Delta\Psi$ which is what we need in the calculation of $T(x)$ from (33) and of q from (34). This can be easily seen by substituting (30) into (28) and setting $\beta_\omega \to 0$. If we wish to know the absolute value of $\Psi(x)$, we have to use the energy equation in the form of (2) as done in the preceding section, but we can no longer obtain the temperature and flux equations as simple as (33) and (34) respectively.

$$B(\omega) = \frac{h\phi_{b\omega 1}[(\sqrt{3})\cosh(\sqrt{3})\beta_\omega + h\sinh(\sqrt{3})\beta_\omega] + (\sqrt{3})h\,\phi_{b\omega 2}}{(3 + h^2)\sinh(\sqrt{3})\beta_\omega + 2(\sqrt{3})h\cosh(\sqrt{3})\beta_\omega} \quad (32)$$

$$A(\omega) = \frac{h}{(\sqrt{3})}[B(\omega) - \phi_{b\omega}(0, \omega)]$$

Integrating (25) twice and using (13) and (14) gives the formal solution of (25) for $T(x)$, as

$$T(x) = [T_1 - (T_1 - T_2)x] - \frac{1}{3N}[\Psi(0) - \Psi(1)]x + \frac{1}{3N}[\Psi(0) - \Psi(x)] \quad (33)$$

where $\Psi(x)$ is given by (28). The term in the first bracket represents the temperature for pure conduction and the rest is that due to radiation. Once $\Psi(x)$ has been found, the heat flux can be calculated by

$$q = 4N(T_1 - T_2) + \tfrac{4}{3}[\Psi(0) - \Psi(1)] + q_a \quad (34)$$

where q_a is given by (23).

The advantage of using the energy equation (3) can be clearly seen from (33) and (34) which are in the same form as those for grey media [9, 11]. Thus, the various approximate methods developed in [9, 11] for the calculation of $T(x)$ and q will apply also for nongrey media with $\varkappa_\omega = \varkappa_\omega(T_0^*, \omega)$. This will be discussed later.

For band emission, β_ω may approach zero at band tips. In the numerical integration of (28) we may assign a small non-zero value of β_ω at

CALCULATED RESULTS

Equations (18) and (21) for the general case were numerically solved by the method of successive approximations on a CDC-6400 computer, for $T_1^* = 3000°K$, $T_2^* = 600°K$, $T_0^* = 1500°K$, $k = 5{\cdot}74 \times 10^3$ erg/(s cm°C) with N as parameter. The relation between the spectral absorption coefficient $\varkappa_\omega$ and ω for given values of T^* was obtained from data reported in [10] and partly plotted in Fig. 1. Fifteen unequal increments of ω ranging from 50 to 7500 cm^{-1} were chosen such that $\varkappa_\omega$ goes from a trough to a peak or a peak to a trough, approximately one-half of an absorption–emission band. Note from Tables in [10] that $\varkappa_\omega(T^*, \omega)$ is nonzero everywhere in the ranges of ω and T^* that we are concerned. Totally 225 exact values of $\varkappa_\omega(T^*, \omega)$ were stored in the machine for later interpolation. A linear temperature distribution was taken as the initial approximation for the iteration process. The iteration was stopped when the maximum error of $T(x)$ was 0·1 per cent of the preceding one. For large values of N (say $N > 0{\cdot}1$), three or four iterations were sufficient, requiring about 20 s. For small values of N, for instance $N = 0{\cdot}01$, the

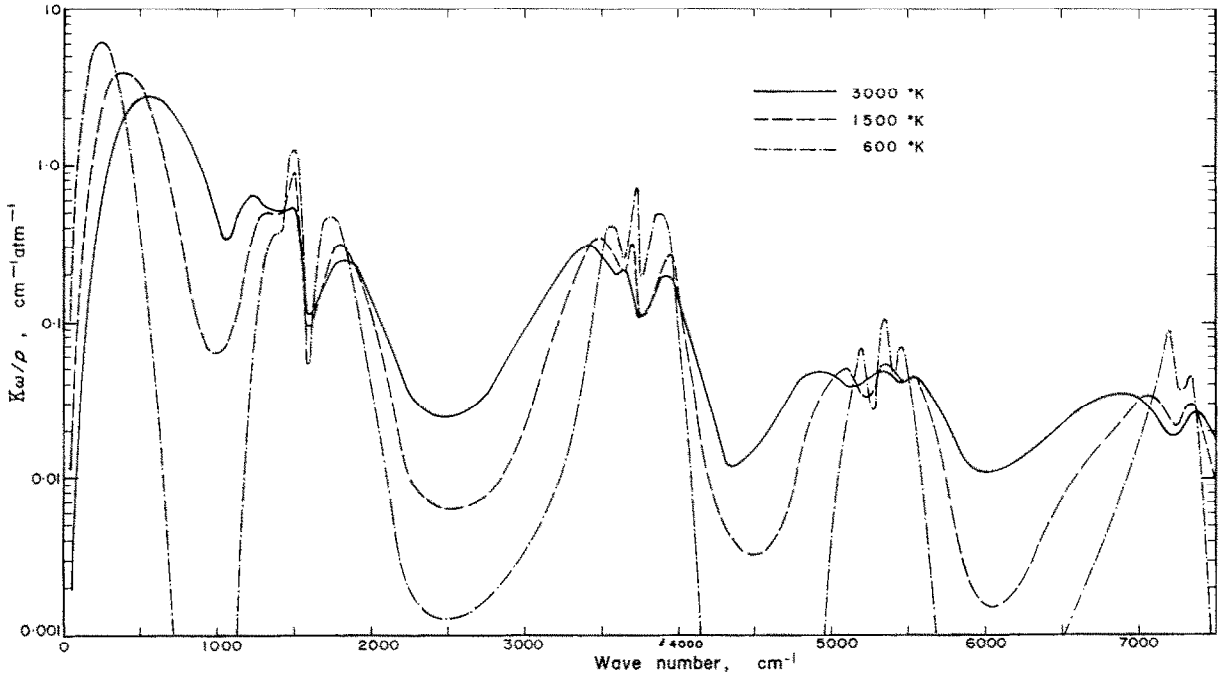

FIG. 1. Spectrum of water vapor [10].

convergence was slow and took about 100 s. Some of the calculated results of the temperature distribution and heat flux for $T_0^* = 1500°K$ are shown in Figs. 2 and 4, respectively.

Equations (30) and (33) for the case of $\varkappa_\omega(T_0^*, \omega)$ were also solved by the method of successive approximations for $T_0^* = 1500°K$. For the evaluation of $\varkappa_\omega(T_0^*, \omega)$, the selection of $T_0^* = 1500°K$ was made on the following basis. When all data of $\varkappa_\omega(T^*, \omega)$ given in [10] were plotted vs. ω with T^* as parameter it was found that $\varkappa_\omega(1500°K, \omega)$ represents approximately the mean value of $\varkappa_\omega(T^*, \omega)$ in the temperature range between 600°K and 3000°K. The calculation for this case is much simpler than that for the general case, and requires much less machine time. Some of the calculated results of the temperature fields and heat fluxes are plotted in Figs. 3 and 4, respectively.

Calculations were also made for a fictitious gas having a constant thermal conductivity and a narrow single-band of absorption–emission which depends only on wave number and pressure, as sketched in the upper left corner of Fig. 5. The purpose of this calculation is to re-examine the inadequacy of the various approximations which were discussed in [4, 5]. For this purpose, we define as in [4] a Planck mean coefficient on the basis of the temperature at the wall $x^* = 0$,

$$\varkappa_P(p) = \int_0^\infty \varkappa_\omega(\omega, p)\, \phi_{b\omega}(T_1^*, \omega)\, d\omega. \qquad (35)$$

As will be seen soon, it is convenient to express equations (24)–(34) in terms of the optical coordinate, $\tau = \varkappa_P(p)\, x$. These equations remain in the same forms provided that x, N and β_ω are replaced, respectively, by τ, N' and γ_ω, and the domain $0 \leqslant x \leqslant 1$ is changed to $0 \leqslant \tau \leqslant l$. Calculations were made for two cases: one of fixed optical thickness, l, and the other of fixed physical thickness, L, with various values of N' which depends on pressure. Note that γ_ω does not vary with pressure and it explains why we

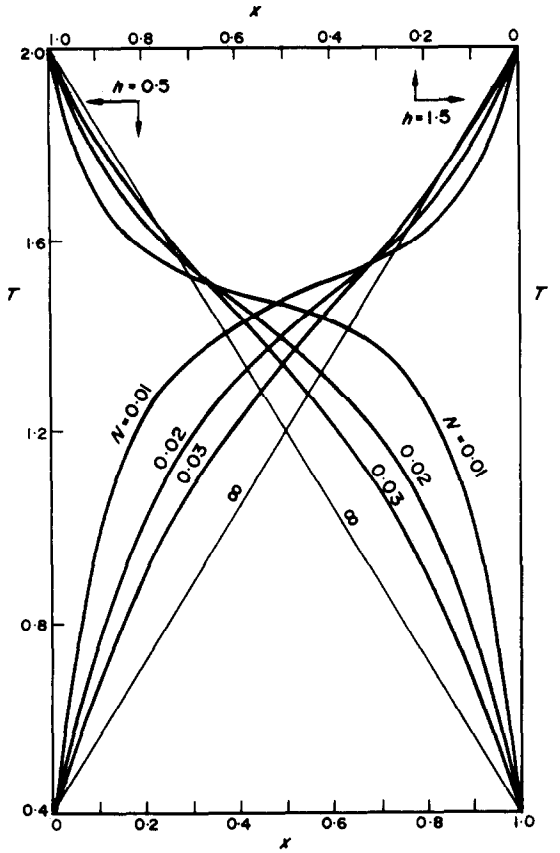

FIG. 2. Temperature distributions in H_2O for $\varkappa_\omega = \varkappa_\omega(T^*, \omega)$, $T_1^* = 3000°K$, $T_2^* = 600°K$ and $T_0^* = 1500°K$.

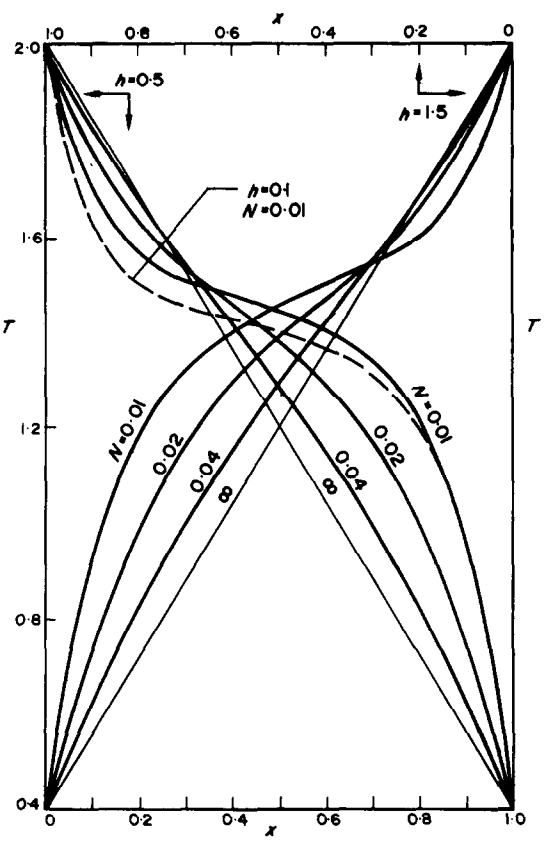

FIG. 3. Temperature distributions in H_2O for $\varkappa_\omega = \varkappa_\omega(T_0^*, \omega)$, $T_1^* = 3000°K$, $T_2^* = 600°K$ and $T_0^* = 1500°K$.

have changed the coordinate from x to τ. Moreover, calculated results for grey cases were mostly reported in terms of the optical coordinate τ. Some calculated results for $T_1^* = T_0^* = 300°K$, $T_2^* = 30°K$ and $k = 1{\cdot}8 \times 10^3$ erg/(s cm°C) are shown in Figs. 5 and 6. Calculations were made for three, arbitrarily assigned, small values of $\varkappa_\omega(p, \omega)$ at band tips and results of $\Delta\Psi$, T and q remained essentially the same.

DISCUSSIONS AND CONCLUDING REMARKS

Temperature curves in Figs. 2 and 3 exhibit the same general feature as those for grey media [11]. Thus, similar discussions as given in [11] for grey cases can be made in regard to the interaction between radiation and conduction and the relative importance between absorption and emission. As N increases, the S-shape of temperature curves becomes less pronounced. For $N \to \infty$, the temperature distribution approaches to that of pure conduction and the heat flux can be calculated by superposition of pure conduction and optically thin radiation,

$$q = 4N(T_1 - T_2) + \frac{1}{2/\varepsilon - 1}(T_1^4 - T_2^4).$$

In fact, this equation gives good approximation for $N > 0{\cdot}1$, as shown by dots in Fig. 4. This can be easily seen from the physical point of view: the larger the value of N the smaller is the value of L and hence the gas layer becomes more transparent.

It is interesting to note from Fig. 4 that heat

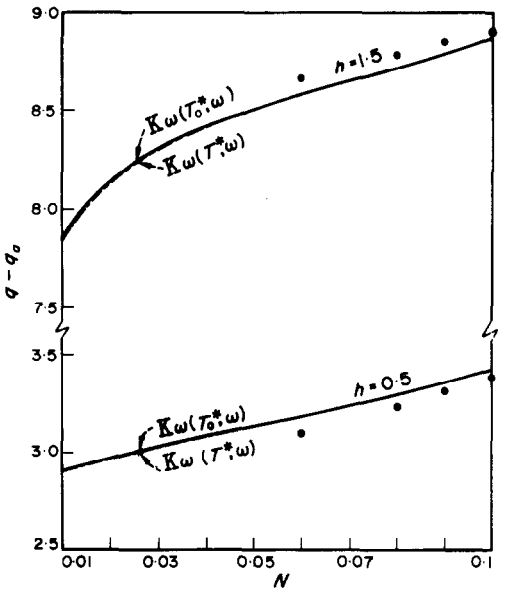

FIG. 4. Heat fluxes in H_2O for $\kappa_\omega(T^*, \omega)$ and $\kappa_\omega(T_0^*, \omega)$ with $T_1^* = 3000°K$, $T_2^* = 600°K$ and $T_0^* = 15000°K$.

fluxes obtained for the two cases, $\kappa_\omega(T^*, \omega)$ and $\kappa_\omega(1500°K)$ are in excellent agreement. In fact, good agreement persists for values of T_0^* in the range from 1500 to 1800°K. The temperature fields as shown in Figs. 2 and 3 are also in reasonable good agreement, with maximum error of 4·2 per cent. When $\kappa_\omega(1800°K, \omega)$ was used, nearly the same results were obtained; the accuracy of temperature fields was somewhat improved, but that of the heat flux slightly reduced. This finding is of great importance in view of the following facts.

The change of the coordinate from x^* to $x_\omega(T^*, \omega)$ is possible only for one-dimensional problems in steady state. In other words, for temperature dependent spectrum, the potential method is restricted only to the solution of steady, one-dimensional problems. However, the change of coordinate from x^* to x, as used in the case of $\kappa_\omega = \kappa_\omega(T_0^*, \omega)$, applies as well for multi-dimensional problems with temperature-dependent spectrum by using the spectrum of a chosen reference temperature, $\kappa_\omega(T_0^*, \omega)$.

Results of grey approximation are also plotted in Figs. 5 and 6 for the fictitious gas. The inadequacy of the grey approximation is clearly shown, but is not as severe as reported in [5]. The total heat flux is under-estimated about 16 per cent for $l = 1$ and $h = 1·5$, and over-estimated about 12·4 per cent for $l = 1$ and $h = 0·5$. For $L = 5·08$ cm and $h = 1·5$, the under estimation increases with the increase of N and hence of pressure, but the difference is considerably less than that found in [5]. The temperature is over-estimated everywhere except in a thin layer near the hotter surface. For the non-grey case, $\kappa_\omega(p, \omega)$, the temperature curve takes on an unpronounced S-shape which shows that the interaction between radiation and conduction is small. Thus, the total heat flux may be calculated by the superposition of pure conduction and radiation.

$$q = \frac{4N'(T_1 - T_2)}{l} + \int_{\Delta\omega} \frac{(\phi_{b\omega 1} - \phi_{b\omega 2})\,d\omega}{2/\varepsilon - 1 + 3\kappa_\omega L/4} + q_a. \qquad (36)$$

Table 1. *Comparison of heat fluxes obtained from* (34), (36) *and* (37)

N'	h	Equation (36)	Equation (37)	Equation (34)
		$l = 1·0$		
0·0266	0·5	0·3910	0·3888	0·4101
	1·5	0·8513	0·9119	0·9230
0·0662	0·5	0·5336	0·5314	0·5517
	1·5	0·9939	1·0545	1·0669
0·1325	0·5	0·7723	0·7700	0·7899
	1·5	1·2325	1·2931	1·3073
0·1986	0·5	1·0102	1·0080	1·0268
	1·5	1·4705	1·5311	1·5441
		$L = 5·08$ cm		
0·0266	0·5	0·5253	0·5193	0·5399
	1·5	1·0666	1·0721	1·0855
0·0662	0·5	0·5017	0·5009	0·5202
	1·5	0·9484	1·0200	1·0330
0·1325	0·5	0·4751	0·4877	0·5044
	1·5	0·8515	0·9852	0·9968
0·1986	0·5	0·4564	0·4802	0·4994
	1·5	0·8008	0·9644	0·9779

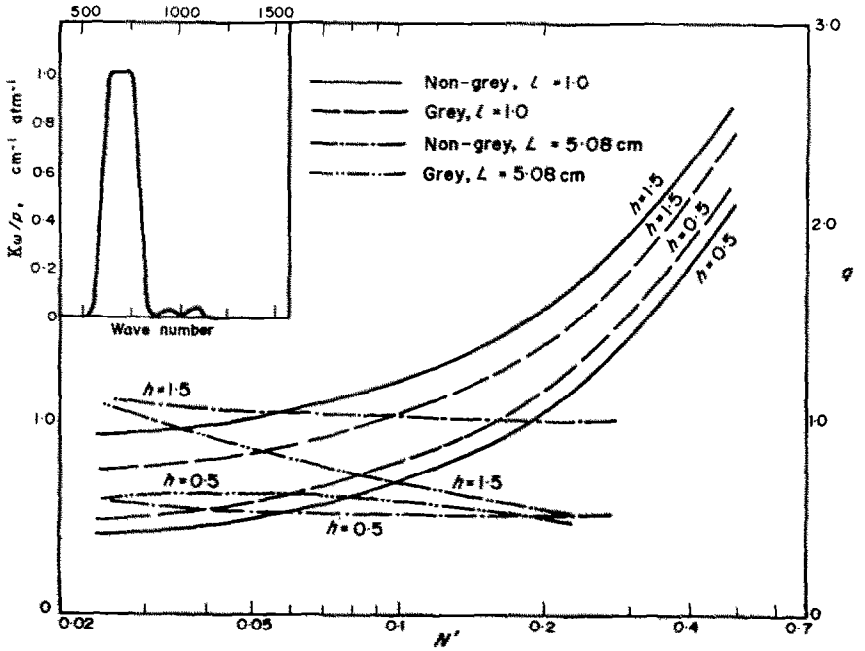

FIG. 5. Temperature distributions in a fictitious gas for $\varkappa_\omega = \varkappa_\omega(p, \omega)$, and $T_1^* = T_0^*$.

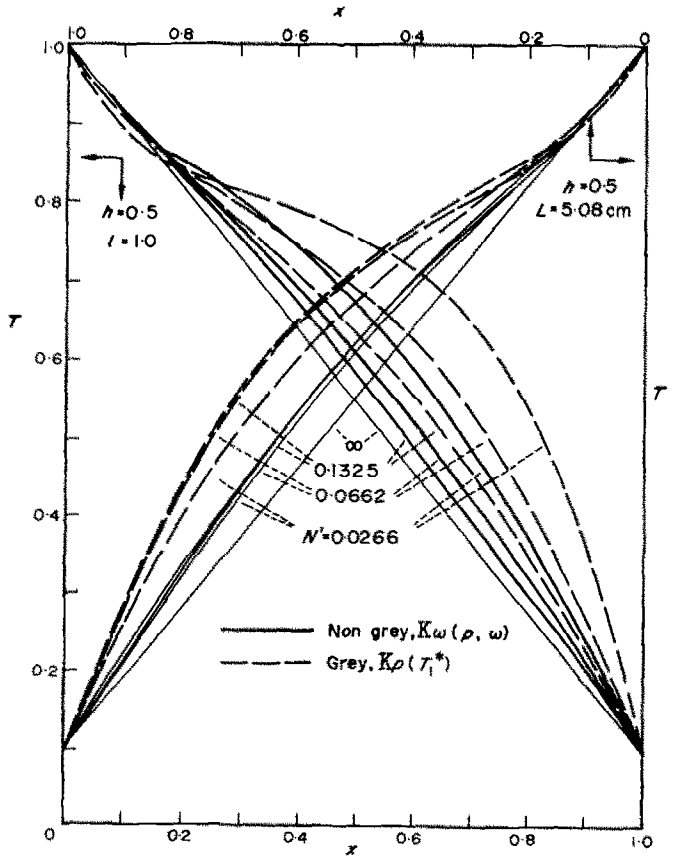

FIG. 6. Heat fluxes in a fictitious gas for $\varkappa_\omega = \varkappa_\omega(p, \omega)$.

If the Planck mean coefficient defined by (35) is used in (36) we obtain

$$q = \frac{4N'(T_1 - T_2)}{l} + \frac{1}{2/\varepsilon - 1 + 3l/4} \times \int_{\Delta\omega} (\phi_{b\omega 1} - \phi_{b\omega 2})\, d\omega + q_a. \quad (37)$$

This is essentially a box model differing from that conventionally defined only by the change of the average absorption coefficient. Calculated results from (36), (37) and (34) are shown in Table 1. It is seen that results obtained from (36) and (37) are in reasonable agreement with those obtained from (34). The inadequacy of grey approximation found from the present study for a narrow single-band of absorption–emission seems to support that reported in [2], but not that in [5].

Calculated results of the radiation-potential drop, i.e. $[\Psi(0) - \Psi(L)]$ not shown in this paper, are essentially independent of N(or N') for a large range of values of N(or N'). Therefore, the total heat fluxes for a large range of values of N(or N') can be calculated by using the potential drop of any value of N(or N') say N_0(or N'_0).

$$q(N) = 4N(T_1 - T_2) + \tfrac{4}{3}[\Psi(0, N_0) - \Psi(L, N_0)] + q_a \quad (38)$$

or

$$q(N') = \frac{4N'(T_1 - T_2)}{l} + \frac{4[\Psi(0, N'_0) - \Psi(L, N'_0)]}{3l} + q_a.$$

This conclusion which has been obtained for grey case [9, 11], holds true also for all nongrey cases investigated here.

Finally, it may be remarked that the potential method has been applied with great facility for the solution of combined heat conduction and radiation in arbitrarily shaped domains by using generalized potentials, or heat potentials [12] and also of heat conduction with change of phase through the use of elementary Green's functions, or fundamental source functions [13]. These results will be reported in the near future. The convergence and uniqueness of solutions (including those in [9, 11] and this paper) will also be discussed then.

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UN TRAITEMENT POTENTIEL DE CONDUCTION DE CHALEUR DANS DE LA VAPEUR D'EAU AVEC UN SPECTRE DÉPENDANT DE LA TEMPÉRATURE

Résumé—Le transfert de chaleur par radiation infra-rouge combinée à la conduction dans une couche de vapeur d'eau est analysé en utilisant la méthode potentielle et des valeurs mesurées du coefficient d'absorption spectral dans le domaine de température compris entre 600°K et 3000°K. On trouve que le problème avec le spectre dépendant de la température peut être traité avec une bonne approximation, comme avec un coefficient d'absorption spectral évalué à une température constante de référence. Ainsi, on établit une méthode approchée de résolution du problème général, au moins pour la vapeur d'eau. Les calculs sont aussi faits pour un gaz fictif ayant une bande unique très étroite d'absorption-émission. De nombreuses approximations discutées par des chercheurs antérieurs sont réexaminées ici.

BEHANDLUNG DER WÄRMELEITUNG IN WASSERDAMPF MIT EINER POTENTIALMETHODE MIT TEMPERATURABHÄNGIGEM SPEKTRUM

Zusammenfassung—Es wird der Wärmeübergang durch kombinierte infrarote Strahlung und Leitung in einer Wasserdampfschicht untersucht, indem eine Potentialmethode und gemessene spektrale Absorptionskoeffizienten in dem Temperaturbereich zwischen 600°K und 3000°K verwendet werden.

Es ergibt sich, dass das Problem mit dem temperaturabhängigen spektrum in guter Näherung so behandelt werden kann, als ob der spektrale Absorptionskoeffizient bei einer konstanten Bezugstemperatur ermittelt wurde. So wurde, zumindest für Wasserdampf, eine Näherungsmethode zur Lösung des allgemeinen Problemes entwickelt. Es wurden auch Rechnungen für ein fiktives Gas mit einem einzigen sehr engen Absorptions-Emissionsband durchgeführt. Verschiedene Näherungen, die von früheren Autoren diskutiert wurden, werden nochmals untersucht.

ПОТЕНЦИАЛЬНЫЙ МЕТОД ИССЛЕДОВАНИЯ ТЕПЛОПРОВОДНОСТИ ВОДЯНОГО ПАРА С ЗАВИСЯЩИМ ОТ ТЕМПЕРАТУРЫ СПЕКТРОМ

Аннотация—С помощью потенциального метода анализируется совместный перенос тепла инфракрасным излучением и теплопроводностью в слое водяного пара при использовании коэффициента спектрального поглощения в диапазоне температур от 600 до 3000°К. Найдено, что задача зависящего от времени спектра может рассматриваться с хорошим приближением как задача с коэффициентом спектрального поглощения, определенным при постоянной исходной температуре. Таким образом, разработан приближенный метод решения общей задачи, по крайней мере, для водяного пара. Расчеты также сделаны для фиктивного газа с очень узкой полосой поглощения-излучения. Пересмотрены различные аппроксимации, которые рассматривались ранее другими исследователями.